MATH 579 Exam 3 Solutions

1. (5-8 points) How many solutions to $w+x+y+z=20$ are there, where w,x,y,z are positive integers?

   Set $w'=w-1, x'=x-1, y'=y-1, z'=z-1$. Then the problem is equivalent to $w'+x'+y'+z'=16$, where w',x',y',z' are nonnegative integers. This last problem has solution $\left(\!\binom{4}{16}\!\right)=\binom{19}{16}=969$.

2. (5-10 points) How many four-digit positive integers have all four digits different?

   9 choices for the thousands digit (not zero), 9 choices for the hundreds (can't repeat), 8 choices for the tens (can't repeat), 7 choices for the units. Hence $9^2\cdot 8\cdot 7=4536$ altogether.

3. (5-10 points) How many four-digit positive integers have the sum of their digits at most 33?

   There are 9000 four-digit positive integers. We will count how many have sum at least 34. They must be permutations of 9999, 9998, 9997, 9988, which have $\binom{4}{0}=1, \binom{4}{1}=4, \binom{4}{1}=4, \binom{4}{2}=6$ permutations respectively. Hence 15 integers have sum that is too large, so our answer is $9000-15=8985$.

4. (5-10 points) How many four-digit positive integers contain the digit 9 and are divisible by 3?

   There are 9000 four-digit positive integers, of which 3000 are divisible by 3. We will count how many do NOT contain the digit 9, then subtract. We have 8 choices for the thousands digit, 9 for the hundreds, 9 for the tens. Then, to make the result divisible by 3, we need to make the sum divisible by 3. That means we must choose from $\{0,3,6\}$ or $\{1,4,7\}$ or $\{2,5,8\}$, depending on the total so far; but in any case there are 3 choices. Hence there are $8\cdot 9^2\cdot 3=1944$ not containing 9. Combining, we have $3000-1944=1056$.
   Note: This problem would be much harder if we required the digit 8 rather than 9.

5. (5-12 points) How many surjective functions are there from [6] to [5]?

   Some two domain elements must map to the same value; there are $\binom{6}{2}=15$ ways to choose those two. Now, we have five domain elements (one of which is a double) and five codomain elements, hence any surjective function must be in fact a bijection and there are $5!=120$ such. Combining, we have $15\cdot 120=1800$ surjective functions.
   Note: This problem would be harder if we made the codomain smaller.